

The following values of the thermophysical characteristics of polymethyl methacrylate were obtained: $\alpha = 1.7 \cdot 10^{-7} \text{ m}^2/\text{sec}$, $c_v = 840 \text{ kJ}/(^{\circ}\text{K} \cdot \text{m}^3)$, $\beta = 62 \cdot 10^{-4} \text{ }^{\circ}\text{K}^{-1}$, $\lambda = 2.4 \text{ J}/(^{\circ}\text{K} \cdot \text{sec} \cdot \text{m})$; they agree well with the known data [3] on the assumption that the reference temperature is 55°C .

NOTATION

α , thermal diffusivity; λ , thermal conductivity; α , heat-transfer coefficient; β , coefficient of thermal expansion; t , time; τ , time intervals; W , rate of thermal expansion; R , radius of specimen; L , length of specimen; $\Delta l_{I, II}$, current elongation of the specimen; ΔL_T , maximum absolute elongation of the specimen; $\Delta l_{1, 2}$, increase in length of the specimen within time τ ; c_v , heat capacity of the specimen; I_0 , Bessel equation of zero order; I_1 , Bessel equation of first order; μ_1 , first root of the Bessel equation; A , constant; b , parameter; P , power of electric heating; Q , total power of thermal losses; v , rate of change of the temperature; $\Delta T_{I, II}$, current values of the temperature.

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DETERMINATION OF THE TEMPERATURE OF PIEZOELECTRIC TRANSDUCERS UPON HARMONIC EXCITATION

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The temperature of a thin piezoceramic shell is determined with a view to the dielectric, mechanical, and piezoelectric losses; a comparison of the results of calculation with the experiment shows that the error does not exceed 15%.

The electrophysical properties of ferroelectric materials used in precision measuring technique, viz., dielectric permeability, polarization, elastic and piezoelectric constants (particularly near phase transitions), depend largely on the temperature.

Energy dissipation and the temperature behavior of dielectrics in an alternating electric field were investigated in [1-4]. In distinction to the solution of the disconnected problem of thermoelectroelasticity for an infinite cylindrical shell [4], the present work submits for a transducer with finite dimensions simpler formulas for calculating the temperature and the heat release.

We examine a cylindrical shell $0 \leq s \leq s_0$ to whose external surfaces the electric potentials $V = V_0 e^{i\omega t}$ are applied (Fig. 1).

The equations of motion of a cylindrical shell have the form [5]

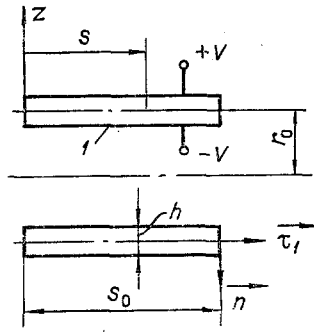


Fig. 1. Calculation schema of a transducer:
1) thermistor.

$$\frac{dN_s}{ds} = -h\rho\omega^2 U_s, \quad \frac{dM_s}{ds} = Q_s, \quad (1)$$

$$\frac{dQ_s}{ds} - \frac{N_\theta}{r_0} + q_n = -h\rho\omega^2 U_z. \quad (2)$$

We adopt the relations of electroelasticity for a transversely polarized piezoceramic shell in the following form [6]:

$$\begin{aligned} N_s &= D_N(\varepsilon_s + \nu\varepsilon_\theta - E_0), \quad N_\theta = D_N(\varepsilon_\theta + \nu\varepsilon_s - E_0), \\ M_s &= D_M\kappa_s, \quad M_\theta = \nu M_s, \quad E_0 = (1 + \nu)d_{31}E_z^{(0)}, \\ D_N &= \frac{h}{s_{11}^E(1 - \nu^2)}, \quad D_M = \frac{h^3\gamma}{12s_{11}^E(1 - \nu^2)}, \quad E_z^{(0)} = -\frac{2V_0}{h}, \\ \gamma &= 1 + \frac{1 + \nu}{2} \frac{K_p^2}{1 - K_p^2}, \quad K_p^2 = \frac{2}{(1 - \nu)} \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T}. \end{aligned} \quad (3)$$

The deformations of the middle surface of the shell are determined via the displacements:

$$\varepsilon_s = \frac{dU_s}{ds}, \quad \varepsilon_\theta = \frac{U_z}{r_0}, \quad \kappa_s = -\frac{d^2U_z}{ds^2}, \quad \kappa_\theta = 0. \quad (4)$$

Let us examine the first lowest oscillation mode, and there the boundary conditions for the system of Eqs. (1)-(3) assume the form

$$U_s|_{s=s_0/2} = 0, \quad N_s = M_s = Q_s = 0 \text{ for } s = 0, s = s_0. \quad (5)$$

With certain values of the ratio h/r_0 (as was shown in [5]), the function U_s can be calculated with the use of the "momentless" theory of shells

$$\begin{aligned} U_s^h &= A \sin k\tilde{s} = \frac{U_s}{r_0}, \quad \tilde{s} = \frac{s}{r_0}, \\ A &= \frac{(1 - \lambda - \nu)E_0}{(1 - \lambda - \nu^2)k \sin \frac{ks_0}{2}}, \quad k^2 = \frac{\lambda}{1 - \frac{\nu^2}{1 - \lambda}}. \end{aligned} \quad (6)$$

Using (6) from the basic system of Eqs. (1)-(4), we find the remaining functions determining the state of stress and strain of the piezoceramic shell.

The equation of thermal conduction was numerically solved on a computer. The calculations showed that the temperature is practically constant across the thickness [4]. The maximum increase of the initial temperature is determined by the formula

$$T_{\max} = \omega \sum_{i=1}^3 \Phi_i / \alpha_i F.$$

The components of the function of energy dissipation on account of dielectric, mechanical, and piezoelectric losses are calculated in the following manner [1-4, 7, 8]:

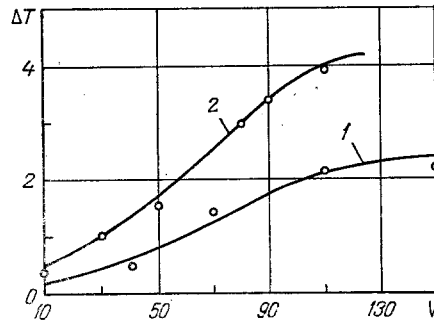


Fig. 2. Dependence of the temperature T ($^{\circ}\text{C}$) on the electric field intensity V (volts) and the frequency: 1) $f = 1$ kHz; 2) 15; dots: experimental data.

$$\Phi_1 = C_{e1} V_0^2 \text{tg } \delta_{dl}, \quad (7)$$

$$\Phi_2 = \frac{0.7\nu E(1 + \text{tg } \delta_m)}{2(1 + \nu)} \left[\varepsilon_s^2 + \varepsilon_\theta^2 + \varepsilon_z^2 + \frac{\nu(\varepsilon_s + \varepsilon_\theta + \varepsilon_z)^2}{(1 - 2\nu)} \right], \quad (8)$$

$$\Phi_3 = -E_z^{(0)} \nu [e_{33}'' \varepsilon_z + e_{31}'' (\varepsilon_s + \varepsilon_\theta)], \quad (9)$$

$$\text{where } \varepsilon_z = s_{13}^E \left(\frac{N_s}{h} + \frac{N_\theta}{h} \right) + d_{33} E_z^{(0)}.$$

In calculations of the function of heat liberation due to elastic and viscoelastic losses in piezoceramic material, the compressive deformations in the second formula $\varepsilon_i =$ ($i = s, \theta, z$) and ε_i^2 are taken with a plus sign, and the corresponding tensile stresses and their squares with a minus sign [7, 8].

On a piezoceramic transducer TsTS-19, which was free of external mechanical forces and had the geometric parameters $r_0 = 14$ mm, $h = 2$ mm, $s_0 = 20$ mm, the steady-state temperature at frequencies of 1 and 15 kHz in the range of change from 10 to 140 V was experimentally measured.

The temperature on the surface of the piezoceramic material was measured with a thermistor MMT-1 with a nominal resistance of 1.6 kilohm which was soldered to the inner electrode of the hollow cylinder by one of its terminals with reliable thermal contact. The thermistor was previously calibrated in a thermostat in the temperature range $20\text{--}32^{\circ}\text{C}$ with the aid of a digital voltmeter V7-16. The error of calibration did not exceed $\pm 0.5^{\circ}\text{C}$. To increase the accuracy in attaining thermal balance between the ceramic parts and the medium in the thermostat chamber, the time between two temperature measurements was chosen within the limits 15-20 min. With the obtained experimental data the graphs in Fig. 2 were plotted.

An analysis of the accuracy of execution of the experiment yields the following formula for the relative error, not exceeding $\pm 8.5\text{--}9\%$:

$$f_{\Delta t} = \pm \sqrt{f_{\Delta R}^2 + f_{\text{tg } \alpha}^2}.$$

In work with a voltmeter V7-16 and a thermistor MMT-1, with $f_{\Delta R} = 2.5\%$ and $f_{\text{tg } \alpha} = 7.9\%$, the experimental error was equal to $f_{\Delta t} \pm 8.3\%$.

For the frequency $\omega = 2\pi \cdot 10^3 \text{ sec}^{-1}$ and $V_0 = 100$ V, the numerical computer calculation yields a maximum temperature increase $T_{\text{max}} = 1.72^{\circ}\text{C}$, in the experiment $T_{\text{max}} = 2^{\circ}\text{C}$. Calculations of the heat release by formulas (7)-(9) showed that in the case under examination heating occurs chiefly on account of internal mechanical losses.

NOTATION

s, θ , meridional and circumferential coordinates of points of the middle surface of the transducer shell; z , radial coordinate; V , electrical potential; r_0 , cylinder radius; N_s, N_θ , meridional and circumferential tensile forces, respectively, in the shell; M_s, M_θ , internal bending moments in the meridional and circumferential directions, respectively; Q_s , transverse shearing forces; h , thickness of the shell; ρ , density; ω , angular oscillation

frequency; U_s , U_z , components of the displacement vector in the direction and normally to the middle surface, respectively; ϵ_s , ϵ_θ , ϵ_z , relative deformations of the piezoceramic material; ν , Poisson ratio; $E^{(0)}$, electric field intensity; d_{31} , piezomodule; s_{11}^E , elastic pliability of the piezoceramic material; C_{e1} , capacitance, v , volume of the transducer; E , Young's modulus; e_{31} , e_{33} , imaginary parts of the complex piezoelectric constants; $f_{\Delta R}$, relative error of measuring the thermistor resistance; $f_{\tan \alpha}$, error of measuring the tangent of the angle of slope of the tangent to the calibration curve; α_t , heat-transfer coefficient; F , area of heat exchange; q_n , uniformly distributed mechanical load normal to the surface of the shell.

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SPECIFIC HEAT IN THE $\text{NH}_4\text{NO}_3\text{-HNO}_3\text{-H}_2\text{O}$ SYSTEM

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The specific heat has been measured in this ternary system. A theoretical study has been made of the effects of temperature and of the electrolyte concentrations (NH_4NO_3 and HNO_3) on the specific heat.

One needs data on the specific heat in the $\text{NH}_4\text{NO}_3\text{-HNO}_3\text{-H}_2\text{O}$ system for technological calculations on the production of water-filled explosives and ammonium nitrate [1].

The specific heat in a ternary electrolyte system (C_{p3} , $\text{kJ}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$) can be calculated theoretically from an equation derived by Zdanovskii from additivity:

$$C_{p3} = C_{p1} \frac{C_1}{X_1} + C_{p2} \frac{C_2}{X_2} \quad (1)$$

To calculate the specific heat of the ternary system containing C_1 and C_2 of each electrolyte component, it is necessary to know the specific heats C_{p1} and C_{p2} for the binary isopiestic solutions. The electrolyte concentrations in the binary isopiestic solutions X_1 and X_2 may be determined graphically from the activities of the water in them. However, the literature carries no data on the water activities in the $\text{NH}_4\text{NO}_3\text{-H}_2\text{O}$ and $\text{HNO}_3\text{-H}_2\text{O}$ binary systems at temperatures above 25°C . It was therefore necessary to measure the specific heats for aqueous solutions of NH_4NO_3 and the ternary system $\text{NH}_4\text{NO}_3\text{-HNO}_3\text{-H}_2\text{O}$ containing up to 80 mass % NH_4NO_3 and up to 30 mass % HNO_3 at temperatures between 20 and 80°C .

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